Antenna Handbook

VOLUME II ANTENNA THEORY

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Contents

Volume II ANTENNA THEORY

Preface

During the past decades, new demands for sophisticated space-age communication and remote sensing systems prompted a surge of R & D activities in the antenna field. There has been an awareness, in the professional community, of the need for a systematic and critical review of the progress made in those activities. This is evidenced by the sudden appearance of many excellent books on the subject after a long dormant period in the sixties and seventies. The goal of this book is to compile a reference to *complement* those books. We believe that this has been achieved to a great degree.

A book of this magnitude cannot be completed without difficulties. We are indebted to many for their dedication and patience and, in particular, to the forty-two contributing authors. Our first thanks go to Mr. Charlie Dresser and Dr. Edward C. Jordan, who initiated the project and persuaded us to make it a reality. After smooth sailing in the first period, the original sponsoring publisher had some unexpected financial problems which delayed its publication three years. In 1988, Van Nostrand Reinhold took over the publication tasks. There were many unsung heroes who devoted their talents to the perfection of the volume. In particular, Mr. Jack Davis spent many arduous hours editing the entire manuscript. Mr. Thomas R. Emrick redrew practically all of the figures with extraordinary precision and professionalism. Ms. Linda Venator, the last publication editor, tied up all of the loose ends at the final stage, including the preparation of the Index. Without their dedication and professionalism, the publication of this book would not have been possible.

Finally, we would like to express our appreciation to our teachers, students, and colleagues for their interest and comments. We are particularly indebted to Professor Edward C. Jordan and Professor George A. Deschamps for their encouragement and teaching, which have had a profound influence on our careers and on our ways of thinking about the matured field of electromagnetics and antennas.

This Preface was originally prepared for the first printing in 1988. Unfortunately, it was omitted at that time due to a change in the publication schedule. Since many readers questioned the lack of a Preface, we are pleased to include it here, and in all future printings.

Preface to the Second Printing

Since the publication of the first printing, we have received many constructive comments from the readers. The foremost was the bulkiness of a single volume for this massive book. The issue of dividing the book into multivolumes had been debated many times. Many users are interested in specific topics and not necessarily the entire book. To meet both needs, the publisher decided to reprint the book in multivolumes. We received this news with great joy, because we now have the opportunity to correct the typos and to insert the original Preface, which includes a heartfelt acknowledgment to all who contributed to this work.

We regret to announce the death of Professor Edward C. Jordan on October 18, 1991.

PART B

Antenna Theory

Chapter 5

Radiation from Apertures

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CONTENTS

Edward V. Jull was born in Calgary, Alberta, Canada. He received a BSc degree in engineering physics from Queen's University, Kingston, Ontario, in 1956, a PhD in electrical engineering in 1960, and a DSc (Eng.) in 1979, both from the University of London, England.

In 1956–57 and 1961–72 he was a research officer in the microwave section and later the antenna engineering section of the Division of Electrical Engineering of the National Research Council of Canada Laboratories in Ottawa. During 1963–65 he was a guest worker in the Electromagnetics Institute of the Technical University of Denmark and the Microwave Institute of the Royal Institute of Technology, Stockholm, Sweden. In 1972 he joined the University of British Columbia, Vancouver, Canada, where he is now a professor in the Department of Electrical Engineering.

In 1964 Dr. Jull was a joint winner of the IEEE Antennas and Propagation Society Best Paper Award. He has been chairman of Canadian Commission VI for the International Union of Radio Science (URSI), chairman of the Canadian National Committee for URSI, an associate editor of *Radio Science*, and an international director of the Electromagnetics Society. He is currently a vice president of URSI and is the author of *Aperture Antennas and Diffraction Theory* (Peter Peregrinus, 1981).

1. Alternative Formulations for Radiation Fields

An aperture antenna is an opening in a surface designed to radiate. Examples are radiating slots, horns, and reflectors. It is usually more convenient to calculate aperture radiation patterns from the electromagnetic fields of the aperture rather than from the currents on the antenna. There are now basically two methods for doing this. Traditionally the pattern has been derived from the tangential electric and magnetic fields in the aperture. This aperture field method is an electromagnetic formulation of the Huygens-Kirchhoff method of optical diffraction. In application it is convenient and accurate for the forward pattern of large apertures. More recently the pattern has also been derived from fields associated with rays which pass through the aperture and rays diffracted by the aperture edges. Its origins can be traced to the early ideas on optical diffraction of Young as more recently formulated by Keller [1] in his geometrical theory of diffraction. It is particularly useful in deriving the radiation pattern in the lateral and rear directions and is described in Chapter 4.

This chapter deals only with the derivation of radiation patterns from the tangential fields in the aperture. Two methods of formulating the radiation integrals are given in this section. They lead to the same result but differ in their concepts of radiation from apertures.

Plane-Wave Spectra

This approach has the advantages of conceptual simplicity for the radiative fields and completeness in its inclusion of the reactive fields of the aperture [2–5]. It uses the fact that any radiating field can be represented by a superposition of plane waves in different directions. The amplitude of the plane waves in the various directions of propagation, or the spectrum function, is determined from the tangential fields in the aperture. This spectrum function is the far-field radiation pattern of the aperture for radiation in real direction angles. The reactive aperture fields are represented by the complex directions of propagation in the total spectrum function.

The method is most appropriate for planar apertures. If the aperture lies in the $z = 0$ plane of Fig. 1 and radiates into $z > 0$, components E_x and E_y of the electric field in $z \geqq 0$ can be written in terms of their corresponding spectrum functions P_x and P_y as

$$\begin{Bmatrix} E_x(x,y,z) \\ E_y(x,y,z) \end{Bmatrix} = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \begin{Bmatrix} P_x(k_x,k_y) \\ P_y(k_x,k_y) \end{Bmatrix} e^{-j(k_x x + k_y y + k_z z)} \, dk_x \, dk_y \tag{1}$$

If α, β are the directions of propagation of each plane wave in Fig. 1, then $k_x = k \sin\alpha \cos\beta$, $k_y = k \sin\alpha \sin\beta$, and $k_z = k \cos\alpha$ are the components of the propagation vector **k** for each plane wave. It is necessary to specify

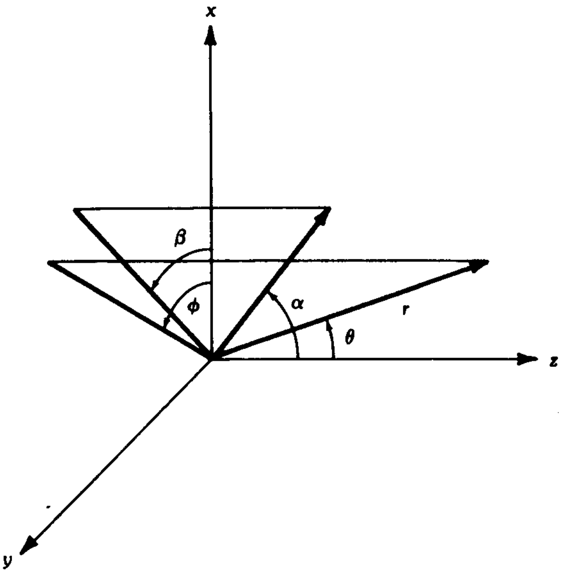

Fig. 1. Radiation from an aperture in the $z = 0$ plane: a plane wave radiates in a direction defined by the angles α and β, and the coordinates of a far-field point are (r, θ, ϕ).

$$k_z = \begin{cases} \sqrt{k^2 - (k_x^2 + k_y^2)} & \text{when } k_x^2 + k_y^2 \leqq k^2 \\ -j\sqrt{(k_x^2 + k_y^2) - k^2} & \text{when } k_x^2 + k_y^2 > k^2 \end{cases} \tag{2}$$

so for real angles α, β (with $k_x^2 + k_y^2 \leqq k^2$) there is radiation, and for complex angles α, β (with $k_x^2 + k_y^2 > k^2$), the fields decay exponentially outwards from the aperture.

Putting $z = 0$ in (1) and inverting the transforms shows the relation between the spectrum functions and the aperture fields:

$$\begin{Bmatrix} P_x(k_x, k_y) \\ P_y(k_x, k_y) \end{Bmatrix} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \begin{Bmatrix} E_x(x, y, 0) \\ E_y(x, y, 0) \end{Bmatrix} e^{j(k_x x + k_y y)}\, dx\, dy \tag{3}$$

Each plane wave has a z component associated with its x and y components in (1) and their relative magnitudes follow from $\mathbf{k} \cdot \mathbf{E} = 0$ for each plane wave. Thus the total field is

$$\begin{aligned} \mathbf{E}(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} & [(\hat{\mathbf{x}} k_z - \hat{\mathbf{z}} k_x) P_x(k_x, k_y) \\ & + (\hat{\mathbf{y}} k_z - \hat{\mathbf{z}} k_y) P_y(k_x, k_y)]\, e^{-j(k_x x + k_y y + k_z z)}\, k_z^{-1}\, dk_x\, dk_y \end{aligned} \tag{4}$$

To evaluate (4) at large distances from the aperture it is convenient to first convert to spherical coordinates, then apply stationary phase integration to the two double integrals. The stationary point is $\alpha = \theta$, $\beta = \phi$, and the final result for $kr \gg 1$ is

$$\mathbf{E}(r,\theta,\phi) \simeq \frac{je^{-jkr}}{\lambda r}[\hat{\boldsymbol{\theta}}(P_x\cos\phi + P_y\sin\phi) - \hat{\boldsymbol{\phi}}\cos\theta(P_x\sin\phi - P_y\cos\phi)] \quad (5)$$

where

$$\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{Bmatrix} E_x(x,y,0) \\ E_y(x,y,0) \end{Bmatrix} e^{jk(x\sin\theta\cos\phi + y\sin\theta\sin\phi)}\,dx\,dy \quad (6)$$

The far magnetic field follows from

$$\mathbf{H}(r,\theta,\phi) = Y_0\hat{\mathbf{r}} \times \mathbf{E}(r,\theta,\phi) \quad (7)$$

where $Y_0 = \sqrt{\epsilon_0/\mu_0}$ is the free-space wave admittance.

Equation 5 provides the complete far-field radiation pattern from the Fourier transforms (6) of the tangential electric field in the aperture. It is also possible to use instead the tangential magnetic field in the aperture. Then, in terms of the spectrum functions

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{Bmatrix} H_x(x,y,0) \\ H_y(x,y,0) \end{Bmatrix} e^{jk(x\sin\theta\cos\phi + y\sin\theta\sin\phi)}\,dx\,dy \quad (8)$$

the total electric field for $kr \gg 1$ is

$$\mathbf{E}(r,\theta,\phi) \simeq \frac{-jZ_0e^{-jkr}}{\lambda r}[\hat{\boldsymbol{\theta}}(Q_x\sin\phi - Q_y\cos\phi)\cos\theta + \hat{\boldsymbol{\phi}}(Q_x\cos\phi + Q_y\sin\phi)] \quad (9)$$

where $\simeq$ means asymptotically equal and $Z_0 = \sqrt{\mu_0/\epsilon_0}$ is the free-space wave impedance.

An equivalent result in terms of both electric and magnetic tangential components of the aperture field is half the sum of (5) and (9), i.e.,

$$\begin{aligned}\mathbf{E}(r,\theta,\phi) \simeq \frac{je^{-jkr}}{2\lambda r}\{&\hat{\boldsymbol{\theta}}[P_x\cos\phi + P_y\sin\phi) - Z_0\cos\theta(Q_x\sin\phi - Q_y\cos\phi)] \\ &- \hat{\boldsymbol{\phi}}[(P_x\sin\phi - P_y\cos\phi)\cos\theta + Z_0(Q_x\cos\phi + Q_y\sin\phi)]\} \quad (10)\end{aligned}$$

in which P_x, P_y and Q_x, Q_y are defined by (6) and (8), respectively. This superposition of electric and magnetic current sources provides a field which satisfies Huygens' principle in that radiation into $z < 0$ is suppressed.

The three expressions (5), (9), and (10) all yield the exact far-field pattern from the exact aperture field integrated over the entire aperture plane. Usually electric fields are more convenient to measure and calculate than magnetic fields, so (9) is rarely used. The choice between (5) and (10) should depend on how well the true boundary conditions are satisfied by whichever approximations are used. For example, it is convenient to assume that the field vanishes in the aperture plane outside the aperture. Then (5) should be used for apertures mounted in a large conducting plane as the boundary conditions on the conductor are rigorously satisfied. For apertures not in a conducting plane, however, (5) with this assumption generally yields less accuracy near the aperture plane than (10).

For apertures which are large in wavelengths the pattern is well predicted away from the aperture plane by all of these expressions. Then the aperture electric and magnetic fields are related by essentially free-space conditions, i.e., $E_x(x,y,0) = Z_0 H_y(x,y,0)$, $E_y(x,y,0) = -Z_0 H_x(x,y,0)$ and $Q_x = -Z_0^{-1} P_y$, $Q_y = Z_0^{-1} P_x$. Equations 9 and 10 become, respectively,

$$\mathbf{E}(r,\theta,\phi) \simeq \frac{je^{-jkr}}{\lambda r}[\hat{\boldsymbol{\theta}}(P_x \cos\phi + P_y \sin\phi)\cos\theta - \hat{\boldsymbol{\phi}}(P_x \sin\phi - P_y \cos\phi)] \tag{11}$$

and

$$\mathbf{E}(r,\theta,\phi) \simeq \frac{je^{-jkr}}{2\lambda r}(1 + \cos\theta)[\hat{\boldsymbol{\theta}}(P_x \cos\phi + P_y \sin\phi) - \hat{\boldsymbol{\phi}}(P_x \sin\phi - P_y \cos\phi)] \tag{12}$$

For small angles of θ, $\cos\theta \cong 1$ and the three expressions (5), (11), and (12) yield essentially identical results in that region of the pattern where their accuracy is highest. All have less precision at angles far off the beam axis, where (12) yields values which are the average of (5) and (11). As (12) is in terms of an aperture electric field which vanishes in the rear ($\theta = \pi$) direction, it is most commonly used for larger apertures, such as horns or reflectors, which are not in a conducting screen.

Equivalent Currents

The fields of sources within a surface S of Fig. 2 can be calculated from the tangential electric and magnetic fields $\mathbf{E}_s$ and $\mathbf{H}_s$ of the sources on S. Alternatively, these surface fields may be replaced by equivalent currents [3, 6, 7]. An electric surface current density $\mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}_s$, where $\hat{\mathbf{n}}$ is a unit vector normally outward from S, represents the tangential magnetic fields. Their contribution to the magnetic vector potential

$$\mathbf{A} = \frac{1}{4\pi}\int_S \mathbf{J}_s \frac{e^{-jkR}}{R}\, dS \tag{13}$$

gives a magnetic field $\mathbf{H} = \nabla \times \mathbf{A}$ and an electric field

$$\mathbf{E} = \frac{1}{j\omega\epsilon_0}\nabla \times \nabla \times \mathbf{A} \tag{14}$$

Similarly, a magnetic surface current density $\mathbf{K}_s = \mathbf{E}_s \times \hat{\mathbf{n}}$ represents the tangential electric fields on S and the resulting electric vector potential

$$\mathbf{F} = \frac{1}{4\pi}\int_S \mathbf{K}_s \frac{e^{-jkR}}{R}\, dS \tag{15}$$

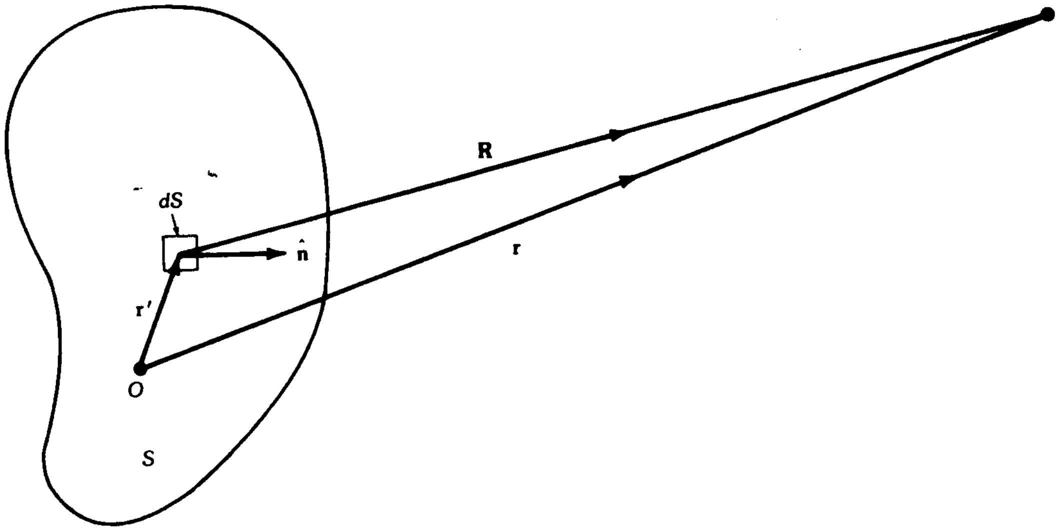

Fig. 2. Coordinates for calculation of radiation from a surface S.

provides an electric field

$$\mathbf{E} = -\nabla \times \mathbf{F} \tag{16}$$

and a magnetic field $\mathbf{H} = \nabla \times \nabla \times \mathbf{F}/(j\omega\mu_0)$.

Contributions from both electric and magnetic current sources give the total electric and magnetic fields outside S:

$$\mathbf{E} = -\nabla \times \mathbf{F} + \frac{1}{j\omega\epsilon_0}\nabla \times \nabla \times \mathbf{A} \tag{17}$$

$$\mathbf{H} = \nabla \times \mathbf{A} + \frac{1}{j\omega\mu_0}\nabla \times \nabla \times \mathbf{F} \tag{18}$$

With this superposition of the fields of electric and magnetic surface current densities radiation into the region enclosed by S is suppressed and Huygens' principle satisfied. There will also be no fields inside S if that region is a perfect conductor. Then $\mathbf{K}_s = 0$ on S and the total electric field outside S can be calculated from (14) with $2\mathbf{J}_s$ in (13).

As discussed previously, under "Plane-Wave Spectra," the fields can be calculated from the tangential electric or magnetic fields on S or from a superposition of the two sets of fields and all three methods yield the same result if the boundary conditions are rigorously satisfied. Usually $\mathbf{E}_s$ and $\mathbf{H}_s$ are approximated by incident fields, and scattered fields on S are neglected. The choice between the three methods should depend on the convenience of accurately approximating the true boundary conditions.

At distances r much larger than the maximum dimension of S in Fig. 2, $R \cong r - \hat{\mathbf{r}}\cdot\mathbf{r}'$ and (13) and (15) become

$$\begin{Bmatrix} \mathbf{A} \\ \mathbf{F} \end{Bmatrix} = \frac{e^{-jkr}}{4\pi r} \int_S \begin{Bmatrix} \mathbf{J}_s \\ \mathbf{K}_s \end{Bmatrix} e^{jk\hat{\mathbf{r}}\cdot\mathbf{r}'}\, dS \tag{19}$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$ and $\mathbf{r}'$ is the vector distance from the origin to dS. Also (14) and (16) become, in the far field in spherical coordinates, respectively,

$$\mathbf{E}(r, \theta, \phi) = -j\omega\mu_0(\hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi) \tag{20}$$

$$= -jk\mathbf{F} \times \hat{\mathbf{r}} \tag{21}$$

Consequently the electric field of electric and magnetic current sources in the far field is

$$\mathbf{E}(r, \theta, \phi) = -j\omega\mu_0(\hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi) - jk\mathbf{F} \times \hat{\mathbf{r}} \tag{22}$$

and the corresponding magnetic fields are given by (7). Again, if electric or magnetic currents alone are used to calculate the total fields from (20) or (21), a factor of 2 must be included in the right side of (19).

With the surface S in the plane $z = 0$ of Fig. 2 and radiation into $z > 0$, $\hat{\mathbf{n}} = \hat{\mathbf{z}}$ in (19), which becomes

$$\mathbf{A} = \frac{e^{-jkr}}{4\pi r}(-\hat{\mathbf{x}}Q_y + \hat{\mathbf{y}}Q_x) \tag{23a}$$

$$\mathbf{F} = \frac{-e^{-jkr}}{4\pi r}(-\hat{\mathbf{x}}P_y + \hat{\mathbf{y}}P_x) \tag{23b}$$

in which P_x, P_y and Q_x, Q_y are defined by (6) and (8). Using (23a) in (20) with a factor of 2 gives (9). Equation 23b in (21) with a factor of 2 gives (5), and (23a) and (23b) in (22) yields (10). Thus the equivalent-current and plane-wave spectrum formulations provide identical results for the radiation fields of an aperture. The equivalent current method is simpler mathematically, but does not account for the reactive fields of the aperture.

2. Radiation Patterns of Planar Aperture Distributions

The expressions (5), (8), and (10) for the radiating far fields of an aperture in terms of the Fourier transforms of the tangential electric and magnetic aperture fields (6) and (8) are exact, but approximations are required in their application.

Approximations

In obtaining the approximations the usual assumptions are the following:

(*a*) The integration limits in (6) and (8) are the antenna aperture dimensions, i.e., fields in the aperture plane outside the aperture are assumed negligible.