

*challenging*  
**MATH**  
**PUZZLES**

**GLEN VECCHIONE**

Illustrated by  
Glen Vecchione & Nina Zottoli

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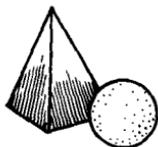
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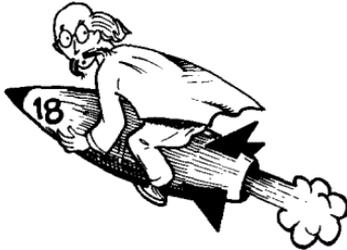
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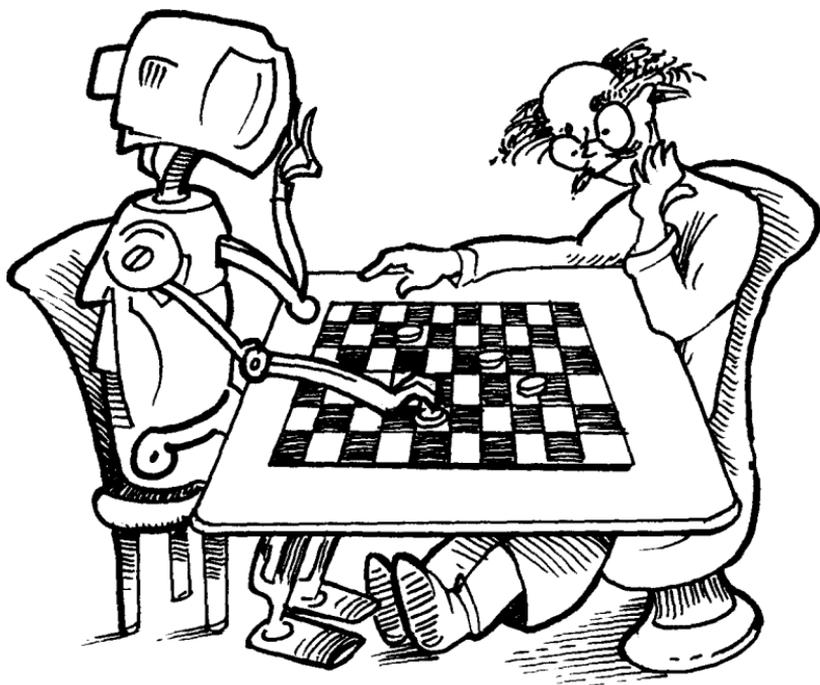


For Briana Vecchione  
*Little Wizard!*



## Chapter 1

# GRID & DOT GAMES



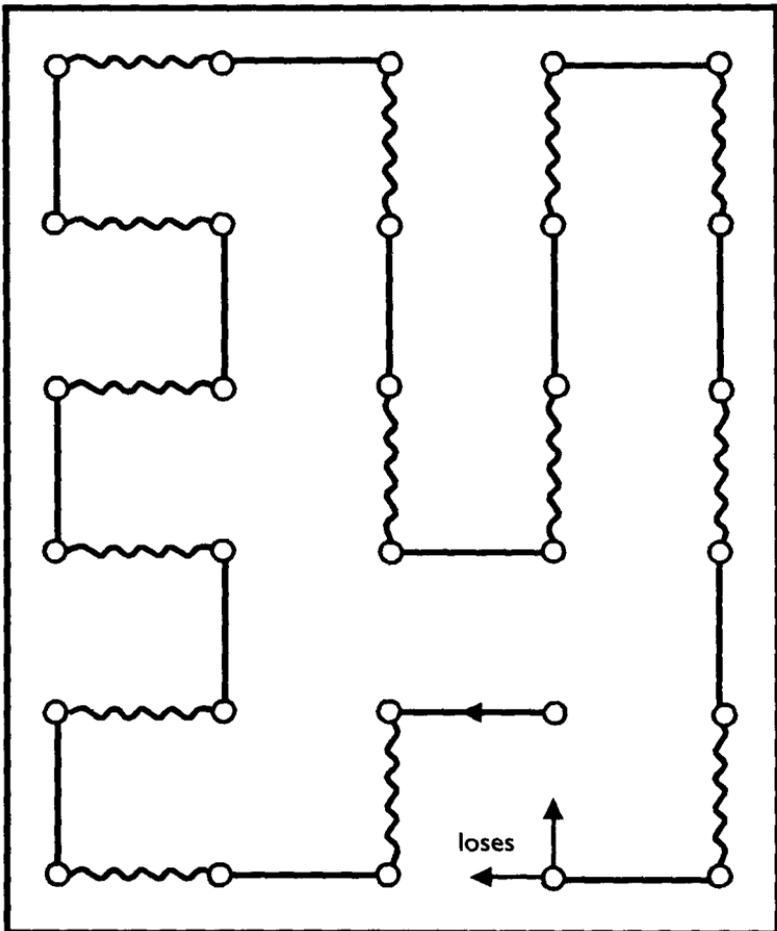
Squiggling Snake • The Cop & the Robber •  
Springing Sprouts • Hare & Hounds • The Ratcatcher  
• Horse Race • Daisy Petals • Black & White

All games are mathematical in that they present both a problem and a method for solving the problem. Board games like checkers or chess depend on the competing player's powers of calculation. Games that use playing cards, number wheels, or dice combine the player's skill with the laws of probability. Most games teach us about what mathematicians call *quantum operations*. This means that players must repeat small, uniform steps to reach a solution within a given framework of laws. So, playing a game of checkers isn't very different from solving a math puzzle.

## SQUIGGLING SNAKE

In this game, players take turns joining dots by a line to make one long snake. No diagonal lines are allowed. Each player adds to the snake at either end, and a player can only add to his opponent's segment, not his own. The first player to make the snake close on itself loses the game.

Here's how an actual game might be played. To tell the two players apart, one player draws straight lines, and the other player draws squiggly lines.

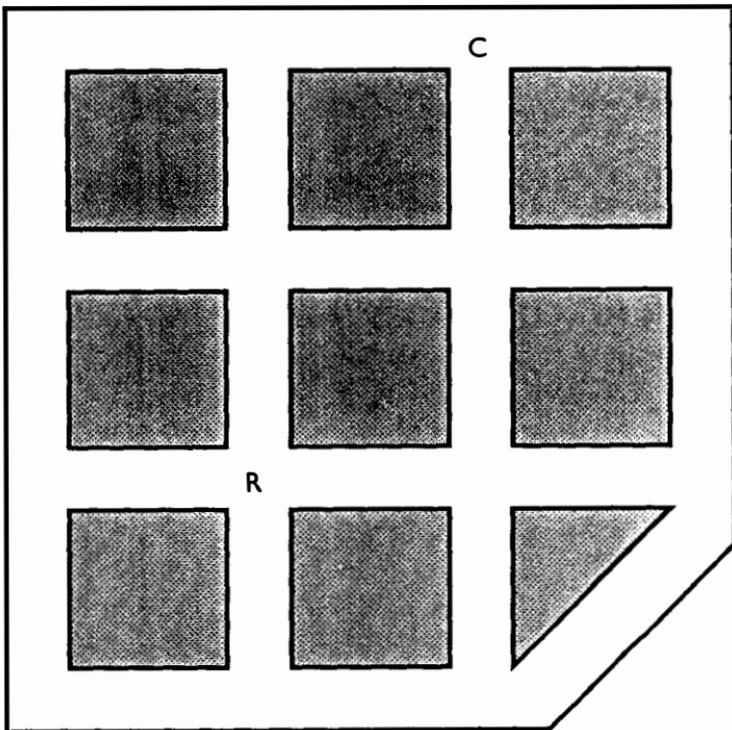


## THE COP & THE ROBBER

On a piece of cardboard or construction paper, draw the game board below including the letters C (cop) and R (robber). This board represents a city grid of several blocks and streets, and the letters indicate the starting position of the cop and robber.

You need two different coins, one for the cop and one for the robber. Start with each coin on its letter. The cop always moves first. After that, robber and cop take turns moving. A player moves a coin one block only, left or right, up or down—that is, from one corner to the next. The cop captures the robber by landing on the robber in one move. To make the game a little more challenging, the cop must capture the robber in twenty moves or less, or the robber wins.

*Hint:* There is a way for the cop to nab the robber. The secret lies in the bottom right corner of the grid.



## SPRINGING SPROUTS

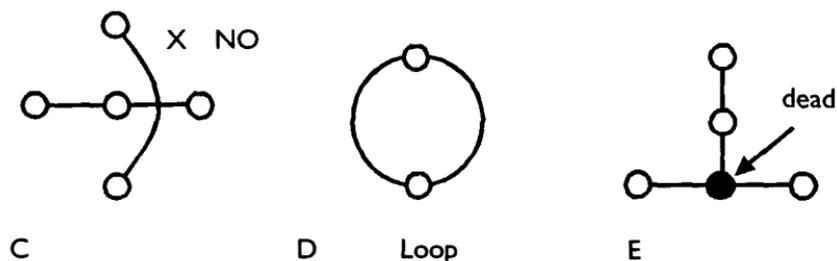
Most mathematical games are played on grids, but some of the newer ones use *topology*, the geometry of flexible lines and surfaces, as a starting point. Springing Sprouts was invented in the 1960s by a mathematician at Cambridge University in England. Not only will you enjoy playing it, but your finished game makes an interesting piece of artwork!

The first player begins by drawing three spots (A). The second player must connect two of the spots with a line (remember, the line can be curved) and then adds a new spot somewhere along that line (B).

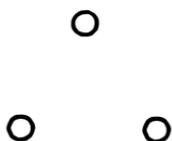


No lines may cross (C), but a player may connect a spot to itself in a loop—as long as he adds another spot (D).

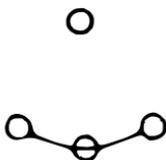
A spot “dies” when three lines lead to it and no more lines can connect to it. To indicate a dead spot, darken it (E).



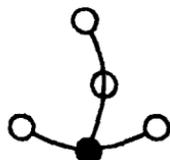
A player wins by drawing the last connecting line so that all the remaining spots are dead and the second player can no longer connect them. Here's a sample game, won by player A in seven moves.



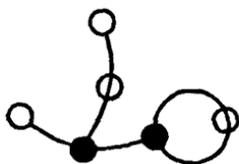
Starting Points



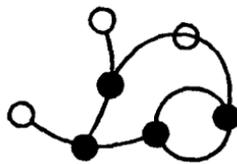
1. A plays



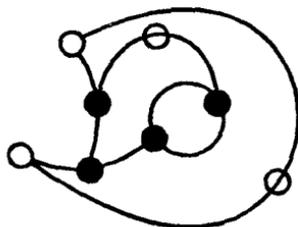
2. B plays



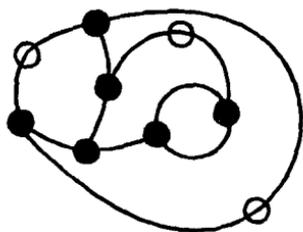
3. A plays



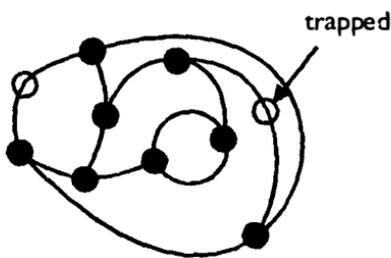
4. B plays



5. A plays



6. B plays



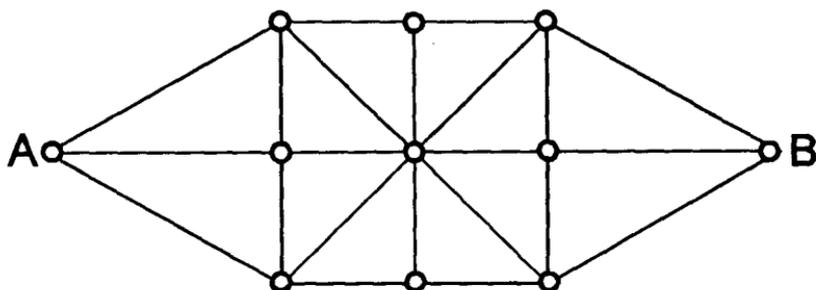
7. A plays

Mathematicians have tried to figure out how many moves it takes for either player to win Springing Sprouts. They've discovered, but not yet proved, that the number lies between twice and three times the number of spots you start with. Starting with three spots, for instance, the game can continue for six to nine moves. Starting with four spots, the game may last for eight to twelve moves, and so on.

## HARE & HOUNDS

Many board games involve “hunting.” This one was a favorite of Victorian schoolchildren. Games like this teach us about geometrical figures called *trapezoids*, which are four-sided figures with only two sides parallel. The corners of trapezoids, or *vertices*, also play an important part in this game.

Use a ruler to draw the board on a piece of paper or cardboard. At every place where the lines meet in a corner, draw a small circle. Draw a large A at the left side of the board and a large B at the right side.

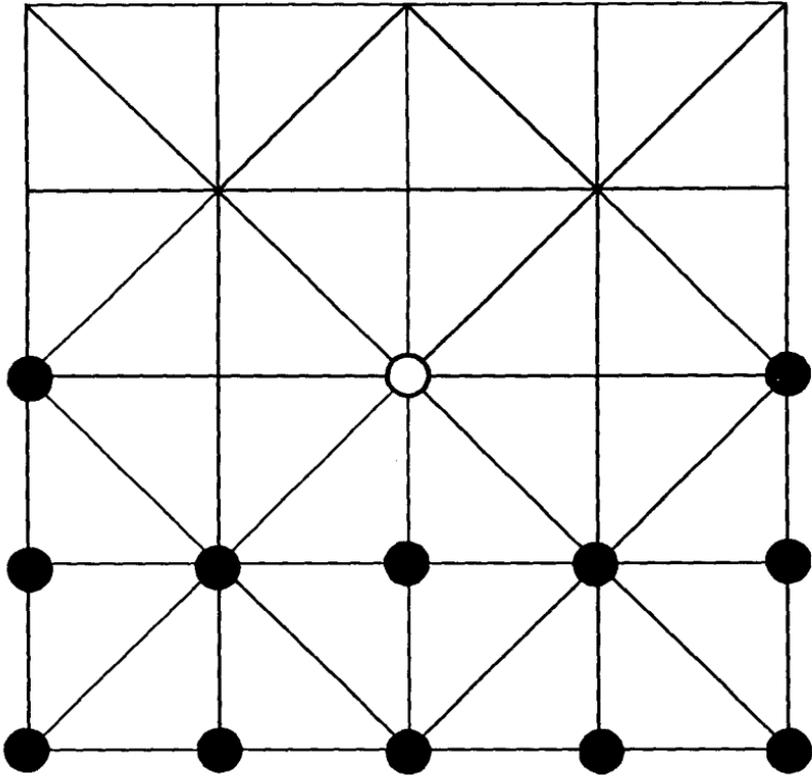


Player #1 has one coin, representing the Hare, and Player #2 has three coins, representing the Hounds. The game starts with Player #1 placing his Hare on circle A. Player #2 follows by placing one of this Hounds on any other circle. He will use the next two turns to place his other Hounds on circles, as the Hare moves from circle to circle, trying to escape.

The Hounds may move in any direction forward (that is, towards circle A) or up and down, but not towards circle B. The aim of the Hare is to reach the safety of circle B while the Hounds, of course, try to block his way and prevent him from moving.

## THE RATCATCHER

This is another version of a strategic chase-and-capture game. One player has twelve Rats, and the other player is the Ratcatcher. The game starts with the thirteen pieces in the positions shown below, the white dot representing the Ratcatcher.

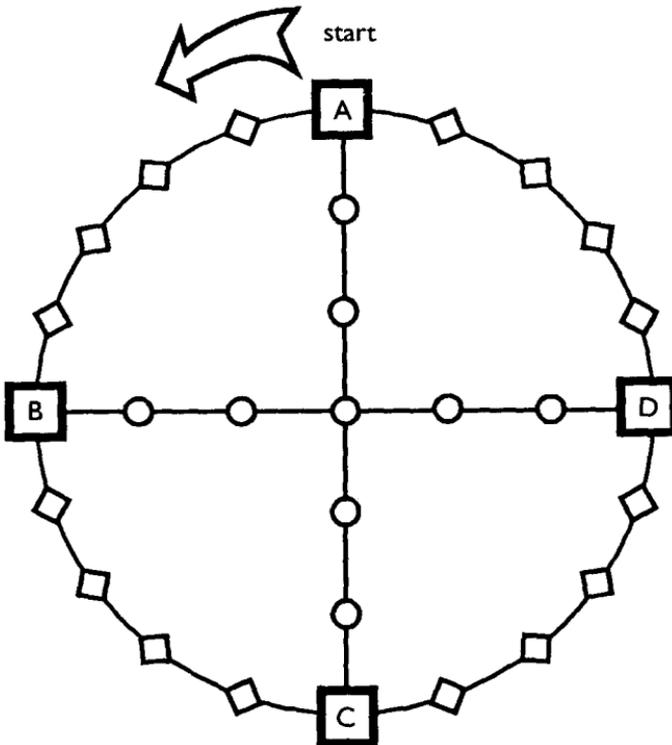


Both Ratcatcher and Rats can move in any direction onto an empty intersection of lines (*vertex*). The Ratcatcher can remove a Rat from the board by jumping over him to an empty vertex on the other side. But the Rats can gang up on the Ratcatcher by surrounding him in such a way that he can neither move nor jump.

## HORSE RACE

In this ancient Korean racing game, *nyout*, players throw one of a pair of dice (a die) to determine the number of moves, then race their buttons, or “horses,” around the circular track illustrated below. Notice that the track has five large squares, sixteen smaller squares, and eight circles.

The players start at square A, and the first player to return to square A wins the game. When a player throws the die, he may only move up to five spaces; a throw of six must be thrown again. If a player’s horse lands on one of the larger squares B, C, or D, the horse may take a shortcut through the circle to reach A.



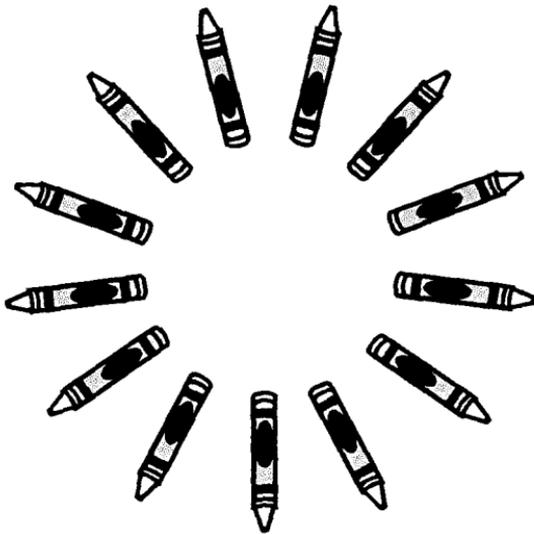
Each player may have up to three horses, but both players must have the same number of horses to ensure a fair game. A player may choose to add another horse instead of moving his original horse.

If one player's horse lands on a square already occupied by another of his own horses, both horses may move together in all the following moves. If one player's horse lands on a square occupied by his opponent's horse, the opponent's horse is removed and must reenter the race.

To win the game, a player must throw the exact number to get each of his horses back to A. It usually takes three or four times around the track before one player's horses win. And your fortunes may reverse at any time!

### DAISY PETALS

On a piece of paper or cardboard, construct a thirteen-petal daisy using crayons for the petals.



Two players take turns plucking either one petal apiece, or two neighboring petals apiece. The player who plucks the last petal wins the game.

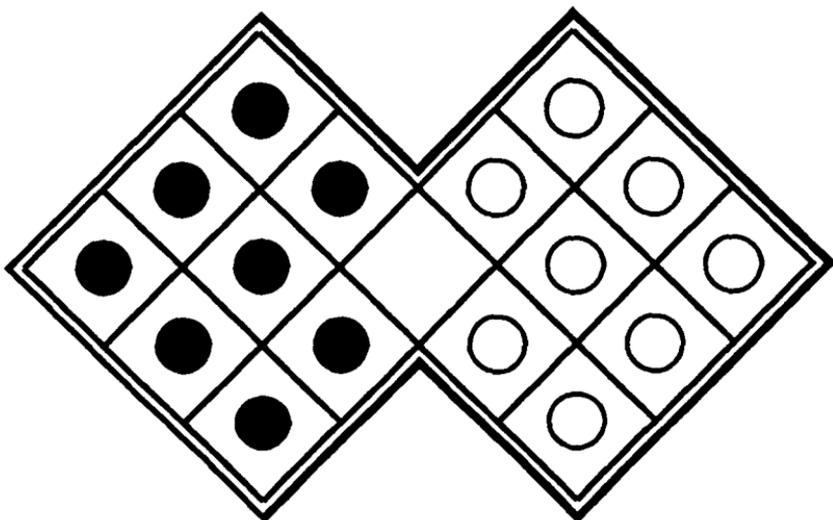
Remember, a player can only take two petals if the petals are neighbors. The player who makes the second move can always win this game if he has a sharp eye and knows something about the principle of *symmetry*—that is, the balance of parts on opposite sides of a line or about a center point.

## BLACK & WHITE

One player alone can enjoy this game, or two players can compete to see who can finish in the least number of moves.

Reproduce the board below with two 3-inch-square pieces of paper, joined at the corner and lined in a grid pattern. In squares on the left side of the board, place eight black pieces (buttons or pennies will do). In the squares on the right side of the board, place eight white pieces.

The object of the game is to exchange the positions of the black and white pieces in the least possible number of moves. You can move a piece by sliding it to a neighboring empty square, or by jumping over a neighboring piece of either color. This can be done in exactly 46 moves, but you can still consider yourself an expert if you finish in 52 moves or less.



## Chapter 2

# SUM OF THE PARTS

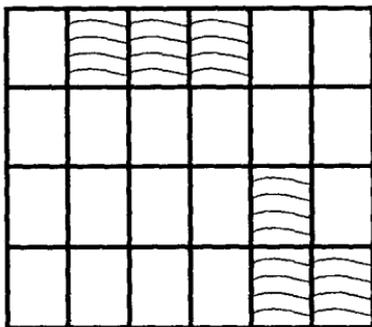


**Stamp Stumper • Broken Dishes • Cut the Pizza •  
Fractured Fractions • Divide the Time •  
Parcels of Land • Four Lines in a Square •  
Count the Blocks • Sides, Edges & Corners •  
Crayon Constructions • Box the Dots**

Mathematicians have two ways of solving puzzles: piecing together small bits of information to understand larger problems and breaking down complicated ideas into simpler parts. For example, *geometry* helps them understand how to combine certain shapes to make larger shapes or how to reduce certain shapes into smaller shapes. *Fractions* help them understand the functions of whole numbers. In each case, breaking something down in order to put it back together again can lead to a valuable understanding of basic mathematical principles.

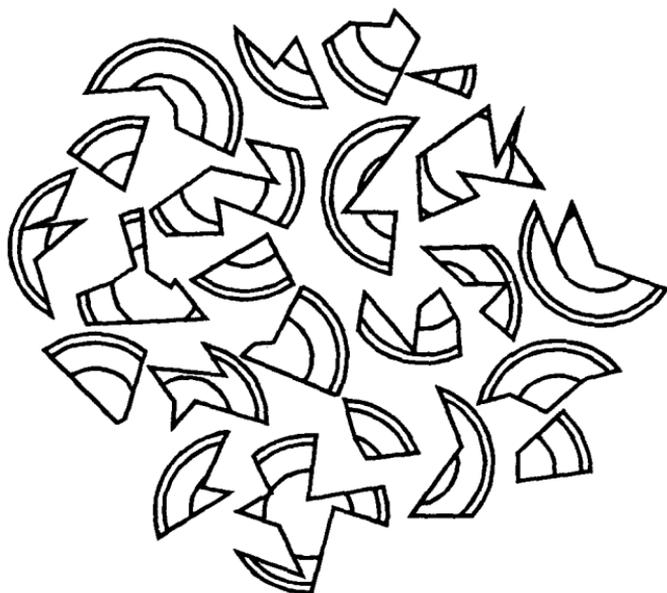
### STAMP STUMPER

Start with a sheet of 24 stamps. Following the diagram below, tear out two sets of three stamps, making sure the stamps of each set remain joined. Tear out a third set, then a fourth set. How many sets, total, can you tear from the sheet?



### BROKEN DISHES

If you could put all the broken pieces back together in this drawing, how many dishes would you have?



## CUT THE PIZZA

A group of eight people walked into a restaurant and ordered a large pizza. The place was very busy, so when the pizza arrived, the waiter wanted to cut it up into eight even pieces as quickly as possible. He did this with only three straight cuts of his knife. Can you figure out how the waiter cut the pizza?



## FRACTURED FRACTIONS

In this puzzle, you must find two-thirds of three-fourths of five numbers.

Find  $\frac{2}{3}$  of  $\frac{3}{4}$  of 12.

Find  $\frac{2}{3}$  of  $\frac{3}{4}$  of 20.

Find  $\frac{2}{3}$  of  $\frac{3}{4}$  of 32.

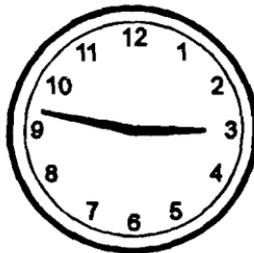
Find  $\frac{2}{3}$  of  $\frac{3}{4}$  of 44.

Find  $\frac{2}{3}$  of  $\frac{3}{4}$  of 52.

Can you discover the trick to doing this quickly?

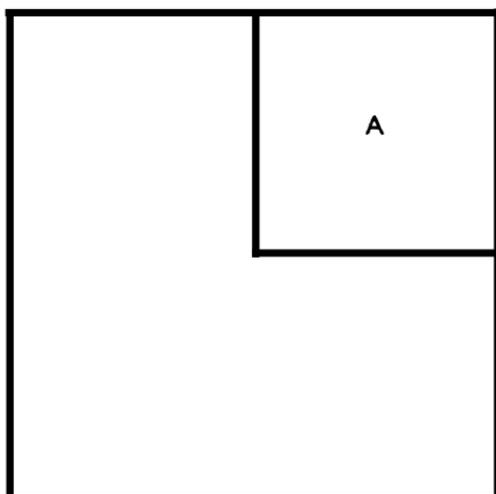
## DIVIDE THE TIME

By drawing only two lines, divide the clockface below so that the numbers in each section add up to the same sum.



## PARCELS OF LAND

A landowner died and left a large, square piece of land to his wife and four sons. His wife received one-fourth of the land (section A), and his sons had to parcel out the remaining three-fourths of land equally. Draw a picture showing how the landowner's sons divided the land. Remember, each of the four sections must be the same size and shape.

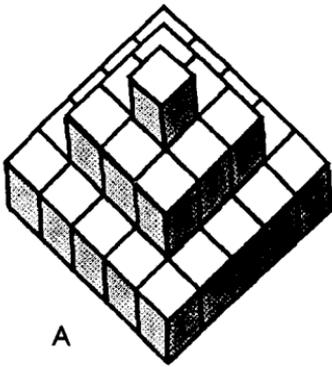


## FOUR LINES IN A SQUARE

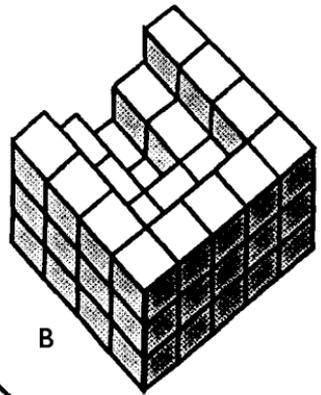
With a pencil and ruler, draw a square. Then draw four straight lines so that each line connects opposite sides of the square. Arrange your four lines so that you divide the square into as many sections as you can. Can you figure out the maximum number of sections you can make with only four lines?

## COUNT THE BLOCKS

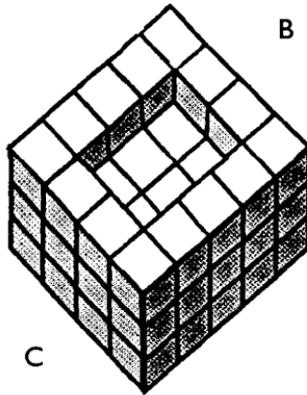
Count the number of blocks in each arrangement. Assume that visible blocks rest on identically shaped hidden blocks, and that every arrangement is solid.



A



B



C

### SIDES, EDGES & CORNERS

In the arrangement below, figure out how many blocks each numbered block touches. Blocks touch if any one of their sides, edges, or corners come in contact.

