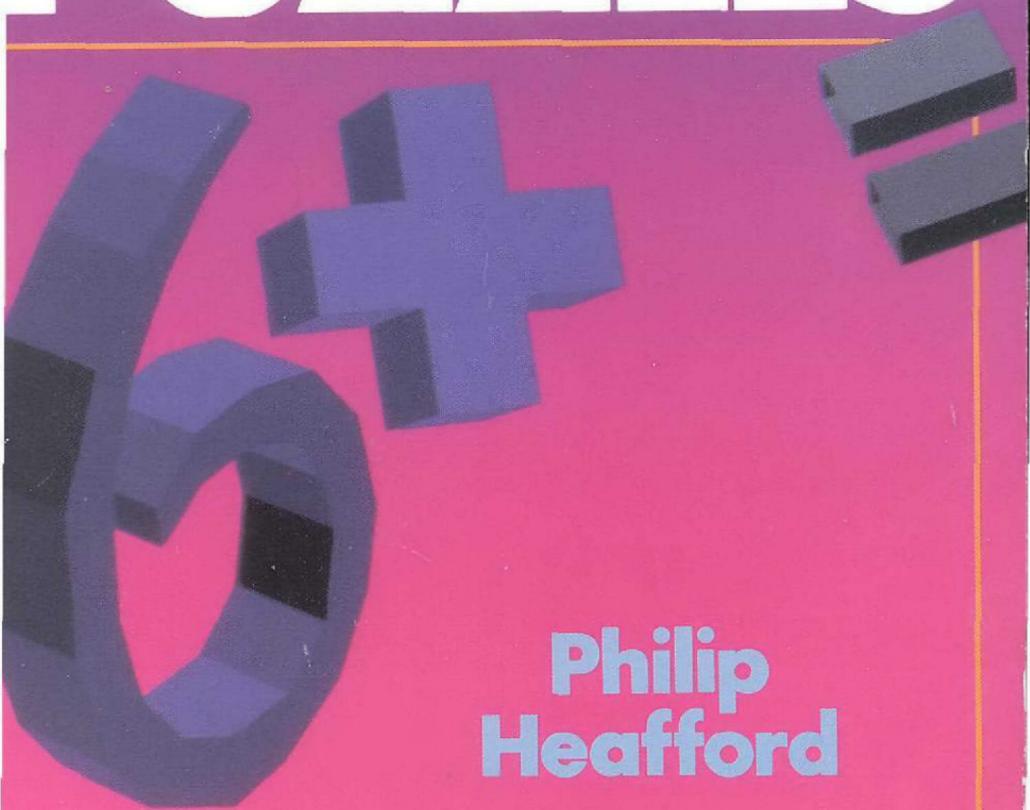


GREAT BOOK OF MATH PUZZLES



Philip
Heafford

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**Philip
Heafford**

 **Sterling Publishing Co., Inc. New York**

To all those who
love to solve a problem

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1. Quickies

Do these numbers ring a bell? For instance, the number 365 would mean only one thing to me, and that is the number of days in a year. Ask someone to test you with this quiz. Six seconds for each question. How many can you get right in the time limit of two minutes for all the questions?

- | | |
|-----------------|---------------------|
| 1. 1,760 | 11. .4771 |
| 2. 2,000 | 12. .4971 |
| 3. 4,840 | 13. 1.6 |
| 4. 640 | 14. 1.414 |
| 5. 1.732 | 15. 1,728 |
| 6. 2.54 | 16. 3-4-5 |
| 7. 3.1416 . . . | 17. 6,080 |
| 8. 366 | 18. $62\frac{1}{2}$ |
| 9. .3010 | 19. 90 |
| 10. 1492 | 20. 88 |

Answers on page 38.

2. The Printer's Nightmare

Before the days of the typewriter, the printer's lot was not always a happy one. Imagine how difficult it must have been for the unfortunate printer trying to set up the type for an arithmetic book when the hand-written manuscript was illegible. One printer overcame this difficulty by putting "stars" for the figures he could not decipher. See if you could have helped him by finding out what the figures really are.

1. Addition:

$$\begin{array}{r}
 \star 22 \star \\
 1 \star \star 1 \\
 \hline
 3489 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 113 \\
 6 \star 4 \\
 14 \star \\
 \star 26 \\
 \hline
 \star 410 \\
 \hline
 \end{array}$$

2. Subtraction:

$$\begin{array}{r}
 4 \star \star 2 \\
 \star 35 \star \\
 \hline
 121 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 6 \star 35 \\
 \star 82 \star \\
 \hline
 4 \star 7 \\
 \hline
 \end{array}$$

3. Multiplication:

$$\begin{array}{r}
 \star 7 \\
 \star \star \\
 \hline
 \star \star \star \\
 \star \star 5 \\
 \hline
 \star \star 91 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 \star \star \star 7 \\
 \star \star \star \\
 \hline
 \star \star \star \star 6 \\
 \star \star 203 \\
 \star 37 \star \star \\
 \hline
 \star \star \star \star \star \star \star \\
 \hline
 \end{array}$$

4. Equations:

$$\begin{aligned}
 5x - 5 &= \star x - 3 \\
 \therefore x &= 2
 \end{aligned}$$

$$\begin{aligned}
 x^2 - 4x &= \star \star \\
 \therefore x &= 7 \text{ or } \star
 \end{aligned}$$

Answers on page 39.

3. Simple? Perhaps!

Can you solve these problems?

1. If five girls pack five boxes of flowers in five minutes, how many girls are required to pack fifty boxes in fifty minutes?
2. A boy has a long cardboard strip 1 inch wide and 48 inches long. It is marked at 1-inch intervals so that he can cut off a series of square inches. If the boy takes one second for each cut, how long will it take to cut the 48 square inches?
3. To move a safe, two cylindrical steel bars 7 inches in diameter are used as rollers. How far will the safe have moved forward when the rollers have made one revolution?
4. A town in India has a population of 20,000 people. 5 per cent of them are one-legged, and half the others go barefoot. How many sandals are worn in the town?
5. Without introducing + signs, arrange six "nines" in such a way that they add up to 100.
6. What is there peculiar about the left-hand side of $50\frac{1}{2} + 49\frac{3}{8} = 100$?
7. A fish had a tail as long as its head plus a quarter the length of its body. Its body was three-quarters of its total length. Its head was 4 inches long. What was the length of the fish?

Answers on pages 39–42.

4. Are You at Home in Rome?

For most of the answers to this quiz you will have to know the Roman figures. As they had no zero to give their numbers a "place value," it must have been very awkward when it came to multiplication!

1. What aid was used by the Romans to help with calculations?
2. The following is cut on a famous monument: MDCCLXXVI. What year does this represent?
3. Write 1789 in Roman figures.
4. What is the largest number you can write using these Roman numerals once each, I,C,X,V,L?
5. What is the smallest number you can write using the same Roman numerals once each, I,C,X,V,L?
6. Without changing to our Hindu-Arabic notation, find the value of $CXVI + XIII + VI + CCLXV$.
7. What Roman numbers of two integers between one and twenty become larger when the left-hand integer is omitted?
8. Was a "groma" used by the Roman merchant, surveyor, cook, or sailor?

Answers on pages 42-44.

5. Easy Teasers

1. During a vacation it rained on thirteen days, but when it rained in the morning the afternoon was fine, and every rainy afternoon was preceded by a fine morning. There were eleven fine mornings and twelve fine afternoons. How long was the vacation?
2. At what time between 7 and 8 o'clock will the two hands of a clock be in a straight line?
3. If $11^3 = 1,331$ and $12^3 = 1,728$, what is the cube root of the perfect cube 1,442,897?
4. A bottle of cider costs 25 cents. The cider cost 15 cents more than the bottle. How much should you receive on returning the bottle?
5. The lengths of the sides of a right-angled triangle measure an exact number of feet. If the hypotenuse is 1 foot longer than the base and the perpendicular is 9 feet long, how long are the sides?
6. A spruce tree when planted was 3 feet high and it grew by an equal number of feet each year. At the end of the seventh year, it was one-ninth taller than at the end of the sixth year. How tall was the tree at the end of the twelfth year?
7. Without doing the actual division state whether 13,972,536 is exactly divisible by 8.
8. A cement mixture costs \$33 a ton. It is composed of Grade A cement at \$36 a ton and Grade B cement at \$24 a ton. How were these two cements mixed?

Answers on pages 44-46.

6. The Triangle Test

A triangle is a geometrical figure bounded by three straight lines and having three angles. Such a definition may be correct, but it gives one the idea that a triangle is a decidedly uninteresting figure. There are many different kinds of triangles and each one has its own interesting peculiarities. From the information given, can you state the names of these triangles?

1. I am readily suggested when you look at a trillium.
2. I appear when a man stands on level ground with his legs straight and his feet slightly apart.
3. I have a special name derived from a Greek word meaning "uneven."
4. I am formed by joining the feet of the perpendiculars from the vertices of a triangle to the opposite sides.
5. The sum of the squares on two of my sides equals the square constructed on my third side.
6. There are at least two of us. We find that our corresponding angles are equal and our sides are proportional.
7. The sides and the diagonals of a quadrilateral are used to construct me.
8. My sides are not straight lines and the sum of my angles is greater than 180° .
9. I have gained the title "pons asinorum" for a certain proposition in Euclid.
10. I am connected with the stars and the zenith.

Answers on pages 46-48.

7. Teasers

1. There are three books, each one inch thick. They stand side by side in order—Volumes I, II, and III. A bookworm starts outside the front cover of Volume I and eats its way through to the outside of the back cover of Volume III. If the worm travels in a straight line, how far does it travel?
2. A man built a cubical house with ordinary windows in all the upright walls. He found whenever he looked out of a window that he was looking south. Where did he build his house?
3. A merchant has two large barrels. The smaller barrel holds 336 liters but is only five-sixths full of wine. He empties this wine into the other barrel and finds that the wine fills only four-ninths of it. How much wine would the larger barrel hold when full?
4. What three curves are produced by making sections of a right circular cone in directions other than parallel to the base?
5. Two men play a card game and the stake is one penny a game. At the end one has won three games and the other has won three pennies. How many games did they play?
6. A number consists of three digits, 9, 5, and another. If these digits are reversed and then subtracted from the original number, an answer will be obtained consisting of the same digits arranged in a different order still. What is that other digit?

Answers on pages 48–49.

8. Some Old & Some New

1. Find a quantity such that the sum of it and one-seventh of it shall equal nineteen.
2. How many guests were present at a Chinese party if every two used a dish for rice between them, every three a dish for broth, every four a dish for meat, and there were 65 dishes altogether.
3. A retired colonel lived a quarter of his life as a boy, one-fifth as a young man, one-third as a man with responsibilities, and thirteen years on pension. How old was he when he died?
4. The fat men in a club outnumber the thin men by sixteen. Seven times the number of fat men exceeds nine times the number of thin men by thirty-two. Find the number of fat and thin men in the club.
5. An explorer grew a beard during his travels. At the end of his journeys, he found that double the length of his whiskers added to its square plus twenty exactly equalled the number of days he had been away. If he had measured the length of his beard in centimeters, and if he had been away 140 days, how long was his beard at the end of his travels?
6. A cathedral tower 200 feet high is 250 feet from a church tower 150 feet high. On the top of each tower is a pigeon. The two pigeons fly off at the same time and at the same speed directly to some grain on the level straight road between the towers. The pigeons reach the grain at the same instant. How far is the grain from the foot of the cathedral tower?

Answers on pages 49–51.

9. Spot the Mistakes

Merely because a statement appears in print it is not necessarily accurate! How often one hears the remark, "I'll show it to you in black and white," as if that is sufficient to decide whether something is true. A mathematician must always be accurate. Are the following statements true or false?

1. The pentagram of Pythagoras is formed by drawing all the diagonals of a regular pentagon.
2. Archimedes was the originator of the well-known puzzle of Achilles and the tortoise.
3. 1:05 p.m. is sometimes written as 1305 hours.
4. The curve in which a uniform cable hangs when suspended from two fixed points is a parabola.
5. A pantograph is a mechanical device for drawing figures similar to given figures.
6. A histogram is a hundred kilograms, and this standard unit is kept at the International Bureau of Weights and Measures at Sèvres, near Paris.
7. A cantilever beam is a beam supported at one end only and extending horizontally.
8. A parameter is an independent variable in terms of which the co-ordinates of a variable point may be expressed.

Answers on pages 51–53.

10. What's My Line?

For purposes of identification certain lines have been given special names, e.g. a tangent, an arc, and a radius. You have to name the line referred to in each of these questions. I . . .

1. join the vertex of a triangle to the mid-point of the opposite side.
2. was said to be the shortest distance between two points.
3. subtend a right angle at the circumference of a circle.
4. am the line so drawn in a circle that the angle between me and a certain tangent is equal to the angle in the alternate segment.
5. "touch" a hyperbola at an infinite distance.
6. cut a circle in two points.
7. join all the points of the same latitude on the earth.
8. am the locus of a point from which the tangents drawn to two given circles are equal.
9. am the essential straight line which, together with the special point or focus, enables points on an ellipse or parabola to be determined.
10. pass through the feet of the perpendiculars drawn to the three sides of a triangle from any point on the circumcircle of the triangle.

Answers on pages 53–55.

11. A Mathematical Mixture

This is a mixed bag of questions. Some are easy and some are hard. There is no connection between them whatsoever. Get busy as the proverbial bee and count how many you can answer correctly. Perfect marks will qualify you for the award of the Pythagorean star which you can draw for yourself. Do you know . . . ?

1. the number of barleycorns in an inch?
2. the instrument used by Sir Francis Drake to find the altitude of the sun and hence the time?
3. the instrument used in the sixteenth century to tell the time at night by observing the constellation Ursa Major?
4. the name of the mathematician who first proved
$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
?
5. the name given to the figure like a five-pointed star often used in the Middle Ages to frighten away witches?
6. what "meter" is used to measure the area contained by a closed plane curve?
7. the name of the solid formed by cutting a pyramid or a cone by two parallel planes?
8. to what use Simpson's rule is put?
9. the common name for a regular hexahedron?
10. how long a clock will take to strike "twelve" if it takes five seconds to strike "six"?

Answers on pages 55-57.

12. Lighter Limericks

1. A dear old Grandpa named Lunn
Is twice as old as his son.
Twenty-five years ago
Their age ratio
Strange enough was three to one.
When does Grandpa celebrate his centenary?
2. Said a certain young lady called Gwen
Of her tally of smitten young men,
"One less and three more
Divided by four
Together give one more than ten."
How many boy friends had she?
3. There was a young fellow named Clive,
His bees numbered ten to the power five.
The daughters to each son
Were as nineteen to one,
A truly remarkable hive!
How many sons (drones) were in the hive?
4. A team's opening batter named Nero
Squared his number of hits, the hero!
After subtracting his score,
He took off ten and two more,
And the final result was a "zero."
How many hits did Nero make?
5. Some freshmen from Trinity Hall
Played hockey with a wonderful ball;
They found that two times its weight,
Plus weight squared, minus eight,
Gave "nothing" in ounces at all.
What was the weight of the ball?

Answers on pages 57–59.

13. A Math Medley

1. What is the name of the small metal frame with a glass or plastic front on which is a fine black line? It is used to facilitate the reading of a slide rule.
2. What is constructed in the same ratio as the following numbers? $24 : 27 : 30 : 32 : 36 : 40 : 45 : 48$.
3. The minute hand of a clock is 7 inches long. What distance does the tip of the hand move in 22 minutes?
4. What curve has been called the "Helen of Geometers"?
5. How can you plant ten tulips in ten straight rows with three tulips in each row?
6. The diameter of a long-playing record is 12 inches. The unused center has a diameter of 4 inches and there is a smooth outer edge 1 inch wide around the recording. If there are 91 grooves to the inch, how far does the needle move during the actual playing of the recording?
7. Two men, Mr. Henry and Mr. Phillips, are appointed to similar positions. One elects to receive a beginning salary of \$3,000 per year with increases of \$600 each year, and the second, Mr. Phillips, chooses a beginning salary of \$1,500 per half-year and an increase of \$300 every six months. Which person is better paid?

Answers on pages 59–61.

14. "C" Gets the Worst of It

Below you will find some problems that were common in arithmetic textbooks fifty years ago. So often Mr. A, Mr. B, and Mr. C appeared, and the unfortunate Mr. C seemed to be the loser, or the person who got the worst of everything! If ever a single person deserves lasting credit from authors it is surely Mr. C. There are no rivals for that honor! Turn the clock back fifty years and solve the following:

1. A field is owned by three people; A has three fifths of it, and B has twice as much as C. What fraction of the field belongs to C?
2. In a mile race A beats B by 20 yards, and he beats C by 40 yards. By how much could B beat C in a mile race?
3. A and B can do a piece of work in ten days; A and C can do it in twelve days; B and C can do it in twenty days. How long will C take to do the work alone?
4. During a game of billiards A can give B 10 points in 50, and B can give C 10 points in 50. How many points in 50 can A give C to make an even game?
5. A, B, and C form a partnership. A furnishes \$1,875, B furnishes \$1,500, and C \$1,250 capital. The partnership makes a profit of \$1,850 in the first year. What should C take as his share of the profit?
6. Pipes A and B can fill a tank in two hours and three hours respectively. Pipe C can empty it in five hours. If all be turned on when the tank is empty, how long will it take to fill?

Answers on pages 61–63.

15. Letters for Numerals

Some simple sums were prepared using the numerals 0 to 9. Then all the numerals were changed to letters. You have to discover the code which was used for the change. You can do this if you look carefully for every possible clue. There is no need to guess. Work these clues methodically, trying each possibility one after the other. There is only one solution to each sum. The code has been changed for each sum. Don't peep at the answers until you have finished and checked your calculation, because the knowledge of one single change will make it too easy and spoil your fun.

1. Addition

$$\begin{array}{r}
 X X X X \\
 Y Y Y Y \\
 Z Z Z Z \\
 \hline
 Y X X X Z \\
 \hline
 \end{array}$$

2. Multiplication

$$\begin{array}{r}
 P N X \\
 N X \\
 \hline
 R N X \\
 N X S \\
 \hline
 Z P N X \\
 \hline
 \end{array}$$

3. Division

$$\begin{array}{r}
 H I L \\
 I L \overline{) P H I L} \\
 I L \\
 \hline
 T I \\
 L S \\
 \hline
 H I L \\
 H I L \\
 \hline
 \dots
 \end{array}$$

4. Division

$$\begin{array}{r}
 Y F Y \\
 A Y \overline{) N E L L Y} \\
 N L Y \\
 \hline
 P P L \\
 P N H \\
 \hline
 N L Y \\
 N L Y \\
 \hline
 \dots
 \end{array}$$

Answers on pages 63–65.